A Fail-Safe Design Approach Based on the Fracture Mechanical Analysis and Epistemic Uncertainty Quantification

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Abstract: The developed method for a fail-safe optimal design of structures is based on a coupled approach of optimization involving a genetic algorithm, the fracture mechanical analysis, and uncertainty analysis enabling the quantification of epistemic uncertainty in the fracture process. The fail-safe structures are intended to retain their functionality even if subjected to certain damage conditions, e.g. a local failure of structural members. In the proposed approach, the failure process is modeled by means of the finite element analysis, employing the concept of discrete fracturing as well as the configurational mechanics based criteria. The investigations on safe failure are enhanced by the quantification of the influence of uncertainties. The uncertainties arising in the failure process of structures, which is governed by crack initiation and growth phenomena are not restricted to variability and randomness. In a structure designed as a system of coupled substructures, the crack initiation position is uncertain but not random, since it results from the boundary conditions change initiated by the occurrence of a certain failure scenario and damage of particular neighbouring structural members. For the modeling of this type of uncertainty, the uncertainty model fuzziness is applied. By means of the proposed method, the optimal design configuration is identified under the consideration of the uncertain crack propagation.

Keywords: fail-safe design, optimization, uncertainty model fuzziness, fracture mechanics

1. Introduction

In civil and mechanical engineering, the fail-safe design concepts are gaining on importance. Thus, for crucial structures, in parallel to standard design procedures aiming at providing sufficient load-bearing capacity, safe failure modes are planned and special design solutions are introduced to enforce an intended structural behavior in case of failure due to unforeseen events as extraordinary loading, impact, fatigue or material defects. Thereby, the main objective is to develop structures, which continue to perform its basic functions even under a certain damage level and are prevented from the catastrophic failure in form of chain reaction like sequential dysfunction of structural components. The fail-safe design strategy constitutes one of three engineering design concepts accounting for failure, next to the safe-life and the damage tolerance design principles (Dilger et al., 2009), (Wood and Engle, 1987), (Cazes, 2013).

The fail-safe design strategies are based upon the application of special design solutions as the redundancy of structural parts, multiple load paths or intentional weak links. Further solutions, considered also in the present contribution, are based on the application of crack arresters, which hinder the crack propagation permitting solely the local failure of a substructure and preventing from damage escalation (Zhang et al., 2009). In parallel to the mentioned particular fail-safe design solutions, some systematic fail-safe design optimization approaches are introduced in (Sun et al., 1976), (Nguyen and Arora, 1982) and (Shechter, 1994). Though, the application of these approaches is limited to truss-like structures and the failure modeling is restricted to the failure of joints. In the present work, a fail-safe design optimization procedure is proposed, which involves failure modeling on the basis of a fracture mechanical solution and is thus applicable to a wider range of structures. In particular, the investigation of the failure process is introduced within a finite element framework by combination of discrete fracturing and configurational mechanics based criteria (Özenç and Kaliske, 2014), (Eshelby, 1951).

Furthermore, the proposed fail-safe design optimization approach is based upon the analysis of uncertainties in the fracture process. An established approach for the consideration of uncertainty of random nature in failure events is the probabilistic fracture mechanics, which is commonly applied for reliability assessment of structures (Rahman, 2001), (Rahman, 2002), (Novák et al., 2005), (Leonel et al., 2010). Further contributions on uncertainty consider a probabilistic fatigue crack growth model (Yang and Manning, 1996), (Besterfield et al., 1991), (Riahi et al., 2010) or investigate the size effect in probabilistic modeling of quasibrittle fracture (Băzant, 2001), (Vorechovský, 2004). In contrary, in the present work, nonstochastic uncertainty model − the model fuzziness (Möller and Beer, 2004), (Möller et al., 2000) is applied for the characterization of the crack initiation phenomenon in a structural system with coupled substructures.

The introduced fail-safe design optimization method is numerically realized as a coupled approach of a genetic algorithm based optimization, the fracture mechanical analysis and the fuzzy analysis. The interaction between the genetic algorithm and the fracture analysis enables to guide the uncertain crack path propagation direction into substructure preserved with crack arresters and thus to identify fail-safe designs, for which only local failures of substructures can occur instead of total failure of the system.

2. Fail-Safe Design Task

The exhibiting of a fail-safe function by a structure is understood as performing according to a predefined safe failure mode defined by a projected, not extensive failure of substructures, while the global system stability is maintained. In addition, the pursuation for the optimality implies that a design configuration is identified, for which the optimal performance within numerous objective functions, also related to safe failure, is expected. The safe failure modes are developed for a structure under the assumption, that the damage of particular substructures occurs in such a manner, that the neighbouring crucial structural elements, which substantially contribute to the global system stability keep performing in an undamaged condition. In the herein introduced fail-safe design strategy, a design concept of a substructure with damage accumulating function is developed, assuming that if this substructure is integrated within a system of coupled substructures it will hinder the progress of the failure mechanism towards crucial structural elements.

For numerical efficiency reasons, the presented fail-safe design optimization task can be solved within a two-step procedure involving: (i) the design optimization of the whole structure and

(ii) the subsequent detailled design of the damage accumulating element. The multi-objective design optimization task of the whole structure, solved in the first step, includes objective functions evaluated in an undamaged condition of the structure, e.g. minimal mass, deformations, production costs as well as in the damaged condition, focusing on obtaining an intended failure path/mode, which ends up in the damage accumulating element. In the second step of the fail-safe design procedure, the design optimization of the damage accumulating element is accomplished, with the initial design configuration identified within the first procedure step. Various loading cases and boundary condition changes of the damage accumulating element are taken into account, which are identified in the analysis of failure modes within the optimization of the whole structure.

The occurrence of a particular failure mode involves a dysfunction of specific substructures, joints or structural members, which are merged to the damage accumulating substructure. The resulting boundary condition changes within the damage accumulating substructure in form of support removal, support displacement or additional loads, lead to stress concentration and may provoke crack nucleation. In order to hinder possible crack propagation, structural elements in form of crack arresters are introduced within the damage accumulating substructure. In this contribution, the second step of the fail-safe design procedure is of main interest and thus a novel concept for the design of the damage accumulating substructure is provided.

For the structure mechanics based design optimization of the damage accumulating substructure, this substructure is considered as a body in the Euclidean space $\mathcal{B} \in \mathscr{E}^3$, surrounded by the boundary Γ, as shown in Figure 1. The body is subjected to body forces $\mathbf{b}$ and surface tractions $\mathbf{t}$, which are assigned to the part $\Gamma_F \in \Gamma$. At the part $\Gamma_b \in \Gamma$, displacement boundary conditions are prescribed. The body includes an initial crack of length γ_{in}, which arises in consequence of local stress concentrations induced by the occurrence of a failure mode. The controlling of partial damage in form of crack propagation and hindering its escalation to further structural components is obtained by the application of crack arresters. In the investigated damage accumulating substructure, n_c crack arresters are assembled, each with a boundary Γ_{Ai}, $i = 1, ..., n_c$ and a geometrical position vector $\boldsymbol{\chi}_{Ai}$ defined with respect to the reference point $P \in \mathcal{B}$. In Figure 1, the body $\mathcal{B}$ with a single crack arrester is presented as an example.

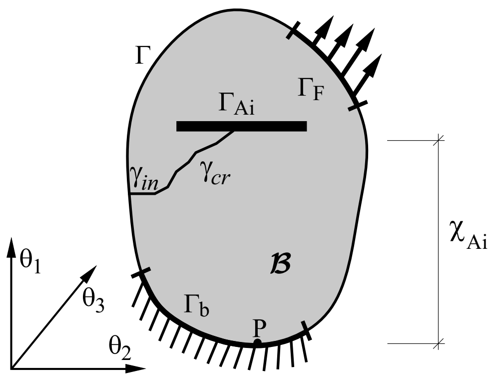

Figure 1. Structure mechanical configuration of the damage accumulating substructure.

The nonlinear multi-objective optimization problem of the damage accumulating substructure is defined as

$$\begin{aligned}
\min_{\mathbf{x}_d \in X \subset \mathbb{R}^n} \quad & \mathbf{f}(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \{f_1(\mathbf{x}_d, \tilde{\mathbf{p}}_a), f_2(\mathbf{x}_d, \tilde{\mathbf{p}}_a), ...f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a), ..., f_m(\mathbf{x}_d, \tilde{\mathbf{p}}_a)\}, \\
& f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \mathcal{K}\left(\tilde{\gamma}_{cr}(\mathbf{x}_d, \tilde{\mathbf{p}}_a, \mathcal{F})\right) \\
\textit{subject to} \quad & G^{\mathcal{M}}(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \nabla_X \cdot \mathbf{\Sigma}^t + \mathbf{B} = 0 \\
& g_k(\mathbf{x}_d, \tilde{\mathbf{p}}_a) \leq 0 \quad k = 1, 2, ..., p, \\
& h_l(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = 0 \quad l = 1, 2, ..., q.
\end{aligned} \tag{1}$$

The major optimization objective of the optimization task in Eq. (1) focuses on the identification of the geometrical configuration $\boldsymbol{\chi}_{Ai}$ of the crack arresters, so that the uncertain crack propagation $\tilde{\gamma}_{cr}(\mathbf{x}_d, \tilde{\mathbf{p}}_a, \mathcal{F})$ is always limited by the crack arrester. Within the definition of the major optimization objective $f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$, $\mathcal{K}$ represents a function utilized for the quantification of the uncertain crack propagation $\tilde{\gamma}_{cr}$, which will be discussed in Section 4. Further objective functions can be considered as well.

In Eq. (1), the objective functions $f_i, (i = 1, 2, ..., m)$ evaluate two types of input parameters, the design variables x_d and the uncertain a priori parameters $\tilde{p}_a$. They are represented by an n-dimensional design vector $\mathbf{x}_d = \{x_{d\,1}, x_{d\,2}, ..., x_{d\,n}\}^T$ and the s-dimensional vector of uncertain a priori parameters $\tilde{\mathbf{p}}_a = \{\tilde{p}_{a\,1}, \tilde{p}_{a\,2}, ..., \tilde{p}_{a\,s}\}^T$ respectively. In the presented optimization problem, $h_l(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = 0$ denote nonlinear equality constraints and $g_k(\mathbf{x}_d, \tilde{\mathbf{p}}_a) \leq 0$ inequality constraints.

Due to the fact, that the major objective function considered in Eq. (1) evaluates the uncertain crack propagation function $\tilde{\gamma}_{cr}(\mathbf{x}_d, \tilde{\mathbf{p}}_a, \mathcal{F})$, the fracture mechanical analysis is applied for the solution of the optimization task. The crack propagation is governed by the energy minimization method on the basis of the configurational forces $\mathcal{F}$. The configurational forces are derived from the material momentum balance equation $G^{\mathcal{M}}$ and then applied for the energy minimization principle to determine the crack growth direction, compare (Özenç and Kaliske, 2014). The material balance equation $G^{\mathcal{M}}$ evaluates forces acting on material inhomogeneities of continuous media in the material space, compare (Eshelby, 1951), (Eshelby, 1975) and (Steinmann, 2000), in contrast to the balance laws formulated in the framework of classical Newtonian mechanics, which involve physical forces. The forces considered within $G^{\mathcal{M}}$ are denoted as configurational or material forces and defined as thermodynamical driving forces. In the optimization task in Eq. (1), the material balance equation $G^{\mathcal{M}}$ yielding the material forces is introduced as an optimization constraint. Thereby, $G^{\mathcal{M}}$ considers a stress measure conjugate to the material forces $\mathcal{F}$, the Eshelby stress tensor $\mathbf{\Sigma}$ as well as the material body force $\mathbf{B}$.

In subsequent sections, various aspects of the optimization task are discussed, as the formulation of the uncertain parameters related to crack initiation and growth and the definition of the objective functions. Finally, a solution concept for the presented fail-safe design optimization problem is provided.

3. Uncertain Crack Initiation and Propagation in Fail-Safe Structures

An important aspect of a fail-safe design strategy is the assessment of potential crack initiation location as well as the prediction of the crack propagation image, taking into account the uncertainties in the fracture processes. A major assumption of the introduced fail-safe design optimization approach is, that in a system with coupled substructures, the crack initiation location in the damage accumulating substructure is uncertain but not random, since it results from the boundary conditions change induced by the occurrence of a certain failure mode and failure of particular neighbouring structural elements. Thus, the nature of the crack initiation process in the context of safe failure of structures designed as a system of coupled substructures differs significantly from e.g. fatigue induced failure of independent systems, which is governed by the crack initiation at randomly distributed material inhomogeneities.

The crack initiation in the damage accumulating substructure is visualized in Figure 2. For the considered body $\mathcal{B}$, the boundary Γ_b with prescribed displacement boundary conditions is divided into three parts Γ_{b1}, Γ_{b2}, Γ_{b3}, whereas each part denotes a support or joint connecting $\mathcal{B}$ with neighbouring structural elements. Should the structure fail according to the first planned failure mode, the failure of support/joint associated with Γ_{b1} within the damage accumulating substructure is expected. Thereby, the consequence of the removal of the boundary part Γ_{b1} is the crack initiation at a position in body $\mathcal{B}$, which is shown in Figure 2 b). The most possible crack initiation location is marked with the black color and the crack initiation locations with gradually decreasing occurrence possibility are indicated by shading in the gray scale. Though, the occurrence of the second planned failure mode, provokes the removal of boundary Γ_{b2} and the uncertain crack initiation in a quite different part of the body $\mathcal{B}$, compare Figure 2 c). For the description of the crack initiation phenomena, the framework of the possibility theory is chosen.

The framework of the fuzzy set theory yields mathematical foundations for the theory of possibility (Dubois and Prade, 1980), (Zadeh, 1965). Especially, the fuzzy set theory based uncertainty model fuzziness is applicable if incomplete, limited statistical or subjective information is evaluated.

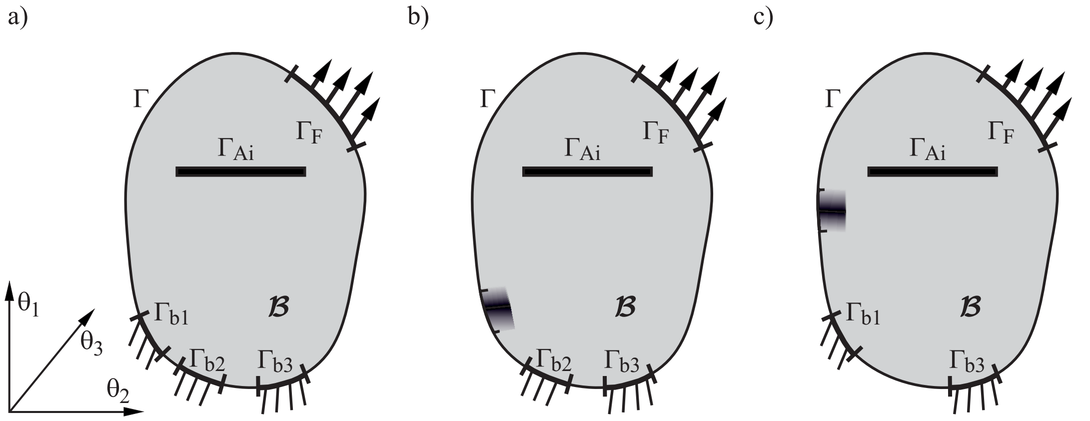

Figure 2. Crack initiation due to the boundary change.

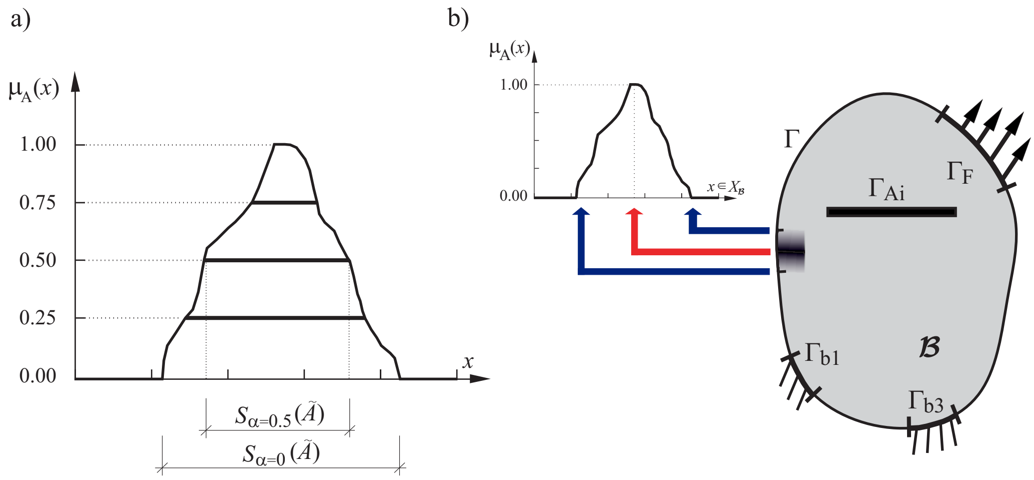

Figure 3. Definition of a fuzzy variable.

Since the information required for modelling the crack initiation position as an uncertain parameter is based on vaque data and/or stems from expert evaluations, its modeling by means of fuzzy sets is reasonable. The fuzzy set $\tilde{A}$ is defined as

$$\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\}. \tag{2}$$

The gradual membership of the elements $x \in X$ to the fuzzy set $\tilde{A}$ is specified by a membership function μ_A, as visualized in Figure 3 a)

$$\mu_A : X \to [0, 1]. \tag{3}$$

The definition of the crack initiation position as a fuzzy set requires the assessment of a set of material points in a subregion of the body $X_{\mathcal{B}} \subseteq \mathcal{B}$ according to the crack initiation potential. The gradual assessment suceeds by means of the membership function μ_A, compare Figure 3 b). In order to provide an interpretation in the context of the possibility theory, a possibility measure Π in the measure space $[X_{\mathcal{B}}, \Sigma, \Pi]$ is assumed, with Σ as the σ-Algebra on $X_{\mathcal{B}}$. Thereby, the possibility measure Π denotes a subjective assessment of the possibility of the occurrence of an event, which is e.g. defined by the crack initiation at a particular point $x \in X_{\mathcal{B}}$. Consider a variable $\tilde{p}_a$ taking values in $X_{\mathcal{B}}$, which is characterized by the possibility distribution function $\pi_d(\tilde{p}_a)$. Thereby, $\pi_d(\tilde{p}_a)$ is defined to be equal to the membership function μ_A of the set $\tilde{A}$

$$\pi_d(\tilde{p}_a) = \mu_A. \tag{4}$$

Thus, an advantageous description of the crack initiation event is obtained since every realization x of the fuzzy variable $\tilde{p}_a$ is quantified by the possibility measure Π and has an assigned degree of possibility $\mu_A(x)$.

The herein presented fail-safe design procedure is based on the solution of the multi-objective optimization task, where the objective functions $f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$, $i = 1, ..., m$ evaluate uncertain input parameter, e.g. the uncertain crack initiation location. Especially, within each objective function $f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$, $i = 1, ..., m$ considered in Eq. (1), a mapping of a design vector $\mathbf{x}_d$, and uncertain parameters vector $\tilde{\mathbf{p}}_a$ onto uncertain structural responses $\tilde{z}_i = f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$ is accomplished by means of the fracture mechanical analysis

$$\begin{aligned} f_i : \mathbb{R}^n \times \mathcal{U}(\mathbb{R}^s) &\rightarrow \mathcal{U}(\mathbb{R}), \\ (\mathbf{x}_d, \tilde{\mathbf{p}}_a) &\mapsto \tilde{z}_i. \end{aligned} \tag{5}$$

In Eq. (5), $\mathcal{U}(\cdot)$ stands for the set of all fuzzy sets, defined respectively, on the space of uncertain parameters $\mathcal{U}(\mathbb{R}^s)$ and on the objective space $\mathcal{U}(\mathbb{R}) \in \mathbb{R}^m$. $\mathbb{R}^n$ denotes the space of design variables. The outputs of the objective functions $f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$, $i = 1, ..., m$ are uncertain structural responses $\tilde{z}_i \in \mathcal{U}(\mathbb{R}^m), i = 1, ..., m$ in form of fuzzy sets. The generation of $\tilde{z}_i$ is accomplished by means of the fuzzy analysis on the basis of the α-level optimization (Möller et al., 2000) and involves the discretization of both, the fuzzy input parameters $\tilde{p}_a$ and fuzzy responses $\tilde{z}_i$ into crisp sets $S_\alpha(\tilde{p}_a)$ and $C_\alpha(\tilde{z}_i)$

$$S_\alpha(\tilde{p}_a) = \{x \in \mathbb{R} : \ \mu_A \geq \alpha\}, \tag{6}$$

$$\tilde{p}_a = (S_\alpha(\tilde{p}_a))_{\alpha \in (0,1]}. \tag{7}$$

$$\tilde{z}_i = (C_\alpha(\tilde{z}_i))_{\alpha \in (0,1]}. \tag{8}$$

The uncertain structural responses $\tilde{z}_i$ can be obtained as fuzzy numbers or may exhibit time τ and space θ dependency and, thus, be characterized by uncertain functions in form of fuzzy processes $\tilde{z}_i(\tau)$, $\tau \in T = \mathbb{R}^4$ and fuzzy fields $\tilde{z}_i(\boldsymbol{\theta})$, $\boldsymbol{\theta} \in T$. The structural response, which is of main interest in this work is the uncertain crack propagation obtained as a fuzzy curve. According to the α-level discretization, the fuzzy crack propagation curve $\tilde{\gamma}_{cr}$ is defined by sets $C_\alpha(\tilde{\gamma}_{cr})$ of trajectories γ_{cr}

$$\tilde{\gamma}_{cr} = (C_\alpha(\tilde{\gamma}_{cr}))_{\alpha \in (0,1]}; \quad C_\alpha(\tilde{\gamma}_{cr}) = \left\{\gamma_{cr} \in \mathscr{E}^3 : \ \mu_z \geq \alpha\right\}. \tag{9}$$

Each trajectory γ_{cr} is equivalent to a deterministic realization of the uncertain propagation curve and corresponds to a deterministic fracture image, compare Figure 4. The set $C_\alpha(\tilde{\gamma}_{cr})$ assembles trajectories, which have at least the assigned membership $\mu_z \geq \alpha$. In addition, every deterministic crack propagation γ_{cr} can be assessed with respect to the possibility of occurrence $\pi_d = \mu_z$. Each trajectory γ_{cr} is defined as a set of n_θ points $\boldsymbol{\theta}_{\boldsymbol{c}}^{\boldsymbol{i}}$ in the body $\mathcal{B}$

$$\gamma_{cr} = \left\{\boldsymbol{\theta}_c^1; ..., \boldsymbol{\theta}_c^i, ..., \boldsymbol{\theta}_c^{n_\theta} \mid \boldsymbol{\theta}_c^i = [\theta_1, \theta_2, \theta_3] \in \ \mathcal{B}\right\}, \tag{10}$$

where the identification of the point $\boldsymbol{\theta}_c^{i+1}$, subsequent to the point $\boldsymbol{\theta}_c^i$, is based on the computation of the material force vector $\mathcal{F}$.

For the uncertain crack propagation $\tilde{\gamma}_{cr}$, the bounding curves $\tilde{\gamma}_{cr}^{\underline{b}}$, $\tilde{\gamma}_{cr}^{\overline{b}}$, which envelope all trajectories γ_{cr}, may be specified

$$\tilde{\gamma}_{cr}^{\underline{b}} = \min_{\boldsymbol{\theta}_c^{\hat{i}j}|_j} [\gamma_{cr} \mid \gamma_{cr} \in C_{\alpha=0}(\tilde{\gamma}_{cr})], \tag{11}$$

$$\tilde{\gamma}_{cr}^{\overline{b}} = \max_{\boldsymbol{\theta}_c^{\overline{\wedge}ij}|_j} [\gamma_{cr} \mid \gamma_{cr} \in C_{\alpha=0}(\tilde{\gamma}_{cr})]. \tag{12}$$

In Eq. (11) and (12), $\boldsymbol{\theta}_c^{\overline{\wedge}ij}$ stands for a position vector projected from the three-dimensional Euclidean space $\mathscr{E}^3$ with dimensions denoted by $i, j, k \in \{1, 2, 3\} \mid i \neq j \neq k$ onto two-dimensional Euclidean space $\mathscr{E}^2$ with dimensions i, j. The bounding curves indicated by black dashed line in Figure 4, are evaluated within the fail-safe design optimization procedure.

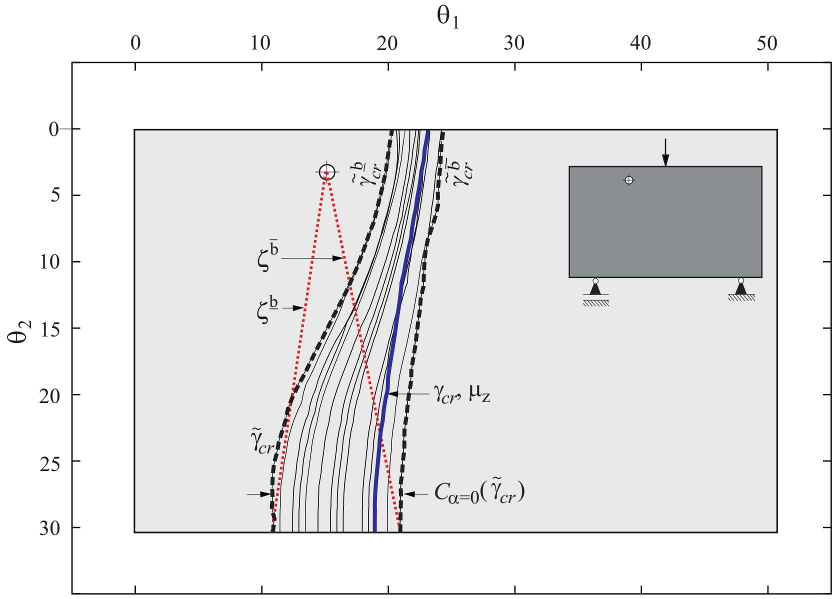

Figure 4. Fuzzy crack propagation curve.

4. Fail-Safe Design Optimization

In the following, the solution of the fail-safe optimal design problem, which is formulated as a the multi-objective optimization task with uncertain (fuzzy) parameters is presented. Since ordering of the objective function outputs is an inherent function of every optimization procedure, the order relations for the uncertain structural responses $\tilde{z}_i$, $\tilde{z}_i(\tau)$, $\tilde{\gamma}_{cr}$ need to be developed. The herein applied order is based on the application of the information reducing measures $\mathcal{M}_j : \mathcal{U}(\mathbb{R}) \rightarrow \mathbb{R}$, (Serafinska et al., 2013), (Graf et al., 2010), (Sickert et al., 2009).

Due to the utilization of information reducing measures $\mathcal{M}_j$ and the application of the scalarization approach for the multi-objective optimization problem in form of the weighted sum method, the objective function vector $\mathbf{f}(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$ in Eq. (1) turns to

$$f(\mathbf{x}_d, \tilde{\mathbf{p}}_a) = \sum_{i=1}^{k} \sum_{j=1}^{l} w_{ij} \mathcal{M}_j(\tilde{z}_i) + \sum_{i=k}^{m-1} \sum_{j=l}^{u-1} w_{ij} \mathcal{K}_{ij}(\mathcal{M}_j(\tilde{z}_i(\tau))) + w_{mu} \mathcal{K}_{mu}(\mathcal{M}_u(\tilde{\gamma}_{cr})), \tag{13}$$

where $\tilde{z}_i = f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$ and w_{ij} are the weighting factors. In Eq. (13), the information reducing measures $\mathcal{M}_j$ are applied to the uncertain structural responses obtained as fuzzy quantities $\tilde{z}_i$, fuzzy functions $\tilde{z}_i(\tau)$ and fuzzy curves $\tilde{\gamma}_{cr}$. Thereby, for fuzzy quantities $\tilde{z}_i$, the measures $\mathcal{M}_j$ defined as the zeroth moment, the variance or the Shannon's entropy quantify the information content, e.g. the uncertainty of $\tilde{z}_i$ and reduce $\tilde{z}_i$ to a crisp value (Sickert et al., 2009). The quantification of the information content of an uncertain function/curve by means of $\mathcal{M}_j$ is equivalent to the identification of deterministic representatives of the fuzzy function/curve, important for a particular optimization objective. An example of representative curves are the deterministic bounding curves $\tilde{\gamma}^{\underline{b}}_{cr}$, $\tilde{\gamma}^{\overline{b}}_{cr}$ of the uncertain crack propagation $\tilde{\gamma}_{cr}$ shown in Figure 4

$$\mathcal{M}_u\left(\tilde{\gamma}_{cr}\right) = \left\{\tilde{\gamma}^{\underline{b}}_{cr}; \tilde{\gamma}^{\overline{b}}_{cr}\right\}. \tag{14}$$

A significant fail-safe design objective is the identification of a design with an optimal position of the crack arresters $\boldsymbol{\chi}_{Ai}$, so that all trajectories γ_{cr} of the uncertain crack propagation curve $\tilde{\gamma}_{cr}$ reach the boundary of the crack arrester Γ_{Ai}. Since all trajectories γ_{cr} of the uncertain crack propagation curve are located between the bounding curves $\tilde{\gamma}^{\underline{b}}_{cr}$, $\tilde{\gamma}^{\overline{b}}_{cr}$, the fail-safe criterion is satisfied if the bounding curves approach the boundary of the crack arrester. Thus, every bounding curve must be assessed with respect to the aspired propagation direction and path. Thereby, the aspired propagation path is defined a priori based on the assumption, that the propagation ends up at the crack arrester, as shown in Figure 4. In Eq. (13), the coincidence between bounding curves $\tilde{\gamma}^{\underline{b}}_{cr}$, $\tilde{\gamma}^{\overline{b}}_{cr}$ and the conjugate aspired crack propagation paths $\zeta^{\underline{b}}$, $\zeta^{\overline{b}}$ is quantified by the function $\mathcal{K}$ utilizing the Euclidean distance metric $d_{\mathscr{E}}$

$$\mathcal{K}_{mu}\left(\mathcal{M}_u\left(\tilde{\gamma}_{cr}\right)\right) = \sum_{l=1}^{n_\theta} P\left[d_{\mathscr{E}}\left(\tilde{\gamma}^{\underline{b}}_{cr}; \zeta^{\underline{b}}\right)\right] + \sum_{l=1}^{n_\theta} P\left[d_{\mathscr{E}}\left(\tilde{\gamma}^{\overline{b}}_{cr}; \zeta^{\overline{b}}\right)\right]. \tag{15}$$

In Eq. (15), n_θ stands for the number of discrete points $\boldsymbol{\theta}_c$ on the bounding curve of the uncertain crack propagation and P is the arbitrarily defined penalty function. The aspired crack curve, e.g. $\zeta^{\underline{b}}$, is specified on the basis of the initial conditions $\zeta^{\underline{b}\,0}$, which are known since they are derived from the definition of the uncertain crack initiation point as a fuzzy quantity $\tilde{p}_a$

$$\zeta^{\underline{b}\,0} = \boldsymbol{\theta}^0_c; \quad \boldsymbol{\theta}^0_c = \min\left[\boldsymbol{\theta}_c \mid \boldsymbol{\theta}_c \in S_{\alpha=0}(\tilde{p}_a)\right]. \tag{16}$$

Further coordinate $\zeta^{\underline{b}\,n_\theta}$ of the aspired crack curve $\zeta^{\underline{b}}$ is prescribed on the boundary of the crack arrester and defined with respect to the location vector of the crack arrester $\boldsymbol{\chi}_{Ai}$

$$\zeta^{\underline{b}\,n_\theta} = \boldsymbol{\theta}^{n_\theta}_c; \quad \boldsymbol{\theta}^{n_\theta}_c = \boldsymbol{\chi}_{Ai}. \tag{17}$$

The interpolation between points $\zeta^{\underline{b}\,0}$ and $\zeta^{\underline{b}\,n_\theta}$ is defined e.g. as a linear function. The aspired crack propagation paths are indicated by a dashed red line in Figure 4. The optimization algorithm identifies an advantageous position of crack arresters, for which the distance between the aspired crack paths and the corresponding bounding curves of the uncertain crack propagation is minimized and all realizations γ_{cr} reach the boundary of crack arresters.

5. Numerical Realization

The numerical realization of the presented fail-safe design optimization concept is based upon a three level procedure implemented as a nested loop approach, compare Figure 5. Accordingly, the optimization constitutes the first level and the outer loop of the numerical procedure whereas the fuzzy analysis establishes the second level and the first inner loop. Within the fuzzy analysis, the fracture mechanical analysis in the finite element framework is executed.

The binary genetic algorithm applied at the optimization level, starts with initialization of the first population of design vectors ξ_ω, $\omega = 0$, and the vector of uncertain a priori parameters $\tilde{\mathbf{p}}_a$.

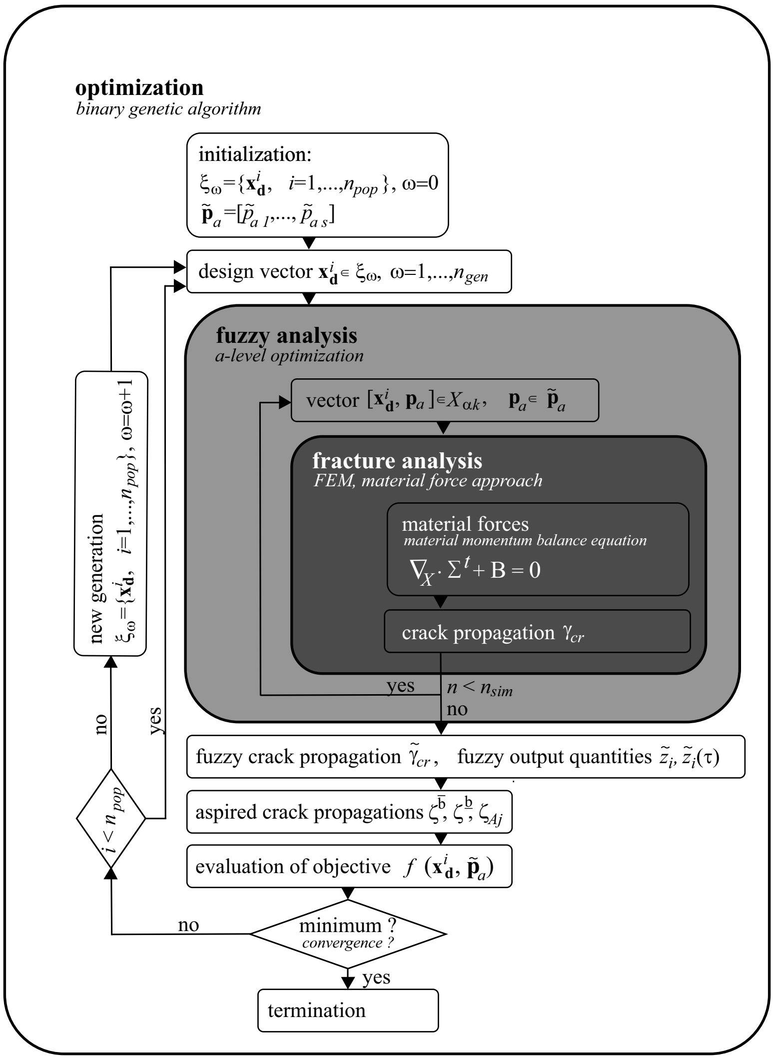

Figure 5. Schematic presentation of the optimization approach.

In the course of the optimization, for each design vector $\mathbf{x}_d^i$, $i = 1, ..., n_{pop}$ in the population ξ_ω, the fuzzy analysis is executed, yielding for every considered objective function $f_i(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$ a fuzzy output quantity $\tilde{z}_i$, $\tilde{z}_i\,(\tau)$ or $\tilde{\gamma}_{cr}$, where $i = 1, ..., m-1$. The computation of fuzzy output quantities within the fuzzy analysis involves the determination of the shape of the membership function μ_z and the support ranges $C_{\alpha=0}(\tilde{z}_i)$. In the present approach, the fuzzy analysis is conducted by means of the α-level optimization procedure, introduced in (Möller et al., 2000). Thereby, the α-level optimization for the determination of the fuzzy crack propagation curve involves the Monte Carlo simulation. Especially, the Monte Carlo simulation is accomplished to identify the crack initiation points. The crack propagation curves resulting from the identified initiation points are evaluated at discrete spatial points to determine the extrema of the spatial dispersion of the uncertain crack propagation at each α-level.

Subsequently, the uncertain responses $\tilde{z}_i$, $\tilde{z}_i\,(\tau)$ and $\tilde{\gamma}_{cr}$ obtained for every considered design $\mathbf{x}_d^i$ are evaluated with the function $f\,(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$ in Eq. (13). Since the formulation of $f\,(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$ is facilitated by the information reducing measures $\mathcal{M}_j$, $f\,(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$ yields a crisp output for a particular design $\mathbf{x}_d^i$. Obtaining of crisp outputs permits the ordering of conjugate designs within the binary genetic algorithm. After the evaluation of the function $f\,(\mathbf{x}_d, \tilde{\mathbf{p}}_a)$, the convergence criterion is verified. If the optimal design is found, the optimization terminates, else the next design is examined.

On the basis of the introduced coupling of the fuzzy analysis, the optimization algorithm and the fracture mechanical analysis, the genetic algorithm learns the features of the uncertain crack propagation and identifies the optimal configuration of the crack arresters.

6. Example

In the present example, the optimal geometrical configuration of crack arresters is determined for a concrete panel with dimensions 50.8 x 30.32 $[cm]$. Thereby, the crack arresting function is exhibited by the openings in the structure, e.g. the service pipes openings, which existence is required anyway. The optimization aims at the identification of the location of four openings $\boldsymbol{\chi}_{Ai}$, $i = 1, ..., 4$, described by six design variables a, b, c, d, e, f as shown in Figure 6.

The crack arrester position vectors $\boldsymbol{\chi}_{Ai}$ are specified with respect to the coordinate system with the origin in the reference point P. The considered design parameters are modelled as discrete variables, compare Table I. The diameter D of all openings is defined by 1.27 $[cm]$ and the magnitude of the initial crack length is set to $\gamma_{in} = 1.254$ $[cm]$. In the model, a linear elastic material characteristic is considered with the modulus of elasticity $E = 38000$ $[MPa]$, Poisson's ratio $\nu = 0.18$ and the fracture toughness $\mathcal{G}_c = 0.5$ $[N/mm]$.The crack initiation position $\tilde{\theta_1^{cr}}$ is considered as an uncertain parameter and modelled as a fuzzy triangular number, compare Figure 6 and Table I. In this example, the crack initiation is a consequence of a particular boundary change, e.g. a removal of an additional support in the bottom of the panel, as depicted in Figure 6.

The fracture analysis is performed with a monotonic displacement based loading at constant increments $\Delta d_z = 0.01$ $[mm]$ on a statically determinate structure, which is obtained after the removal of the additional support. In Figure 7, the crack propagation for the identified optimal design at different stages of the loading and a particular crack initiation position is visualized. At

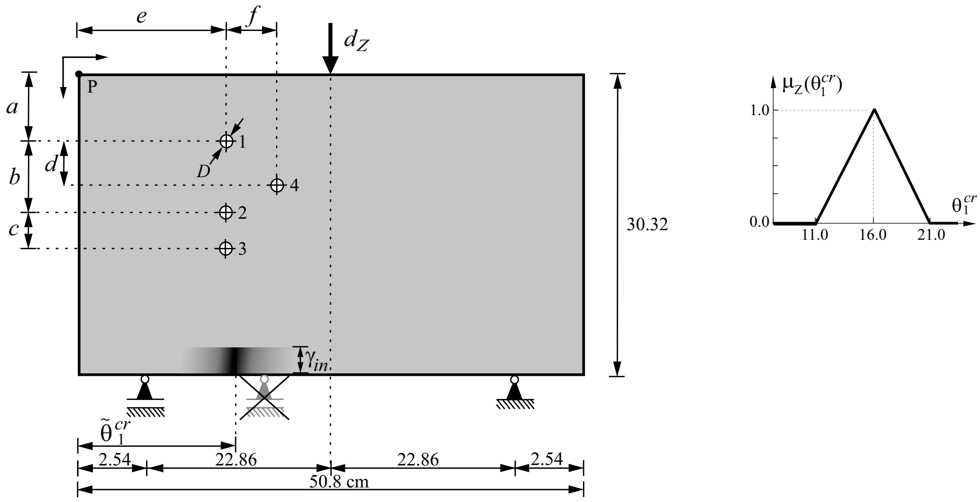

Figure 6. Parametrization of the panel design.

the nodes of the finite element model, the material force vectors are marked, whereas the vector at the crack tip defines the crack driving force.

The optimization accomplished by means of the binary genetic algorithm involves 20 generations of the algorithm and the population size of 25 genomes. The fuzzy analysis is executed considering the discretization of the fuzzy variable $\tilde{\theta_1^{cr}}$ into two α-level sets at $\alpha = 0$ $S_{\alpha=0}(\tilde{\theta_1^{cr}})$ and at $\alpha = 1$ $S_{\alpha=1}(\tilde{\theta_1^{cr}})$. The evaluation of the α-level set $S_{\alpha=0}(\tilde{\theta_1^{cr}})$, enables to account for all possibly appearing crack initiation locations. In addition, the proposed method permits to consider only the crack initiation positions with highest possibility by the evaluation of the α-level sets $S_{\alpha}(\tilde{\theta_1^{cr}})$ with $\alpha \approx 1$.

Table I. Definition of the design variables and uncertain parameters.

Design variables			
	interval	increment	unit
a	$[3.175, 8.255]$	$\Delta a = 2.54$	$[cm]$
b	$[2.54, 5.08]$	$\Delta b = 1.27$	$[cm]$
c	$[2.54, 5.08]$	$\Delta c = 1.27$	$[cm]$
d	$[0.00, 10.16]$	$\Delta d = 2.54$	$[cm]$
e	$[2.54, 27.94]$	$\Delta e = 5.08$	$[cm]$
f	$[2.54, 10.16]$	$\Delta f = 2.54$	$[cm]$
Fuzzy parameter			
$\tilde{p}_{a\,1} = \tilde{\theta_1^{cr}}$	$< 11.0; 16.0; 21.0 >$		$[cm]$

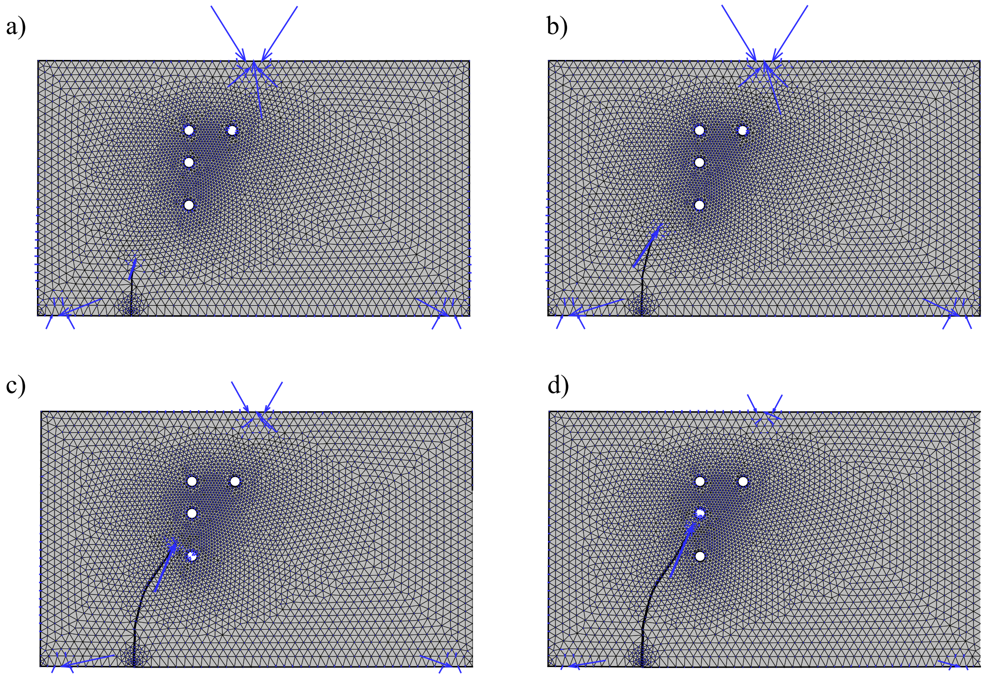

Figure 7. Crack propagation at different stages of the computation a) − d), at applied deformation of 0.70, 0.76, 0.87 and 0.95 $[mm]$ and the nodal material force vectors.

The α-level optimization is performed on the basis of the Monte Carlo simulation with 14 crack initiation locations evaluated for every design. The aspired crack propagation curves are specified as linear functions based on the points $\boldsymbol{\theta}_c^0$ and $\boldsymbol{\theta}_c^{n_\theta}$ available for each considered crack initiation point. The penalty function is defined as $P = [d_{\mathscr{E}}\left(\gamma_{cr\,j}; \zeta_{Ai}\right)]^3$.

In Figure 8, the improvement of designs analyzed in subsequent optimization steps is visualized. For the design evaluated in the initial optimization step, which is presented in Figure 8 a), the disadvantageous configuration of the crack arresters implies that no crack propagation, i.e. no realization of the fuzzy crack propagation curve can be hindered. The enhancement of the fail-safe function is achieved for designs in Figure 8 b), c) analyzed in later optimization stages. In the progress of optimization, the fail-safe optimal design is identified with the crack arrester configuration limiting every crack growth determined by the fuzzy analysis. The optimal design, visualied in Figure 8 d), is characterized by the design variables configuration $a = 8.26$, $b = 3.81$, $c = 5.05$, $d = 0.00$, $e = 17.78$, $f = 5.08\,[cm]$ and the corresponding crack arrester position vectors $\boldsymbol{\chi}_{A1} = [17.78, 8.26]$, $\boldsymbol{\chi}_{A2} = [17.78, 12.06]$, $\boldsymbol{\chi}_{A3} = [17.78, 17.14]$, $\boldsymbol{\chi}_{A4} = [22.86, 8.26]$ $[cm]$. The FE models associated with the design configurations evaluated in the optimization are depicted in Figure 8 as well. Thereby, FE models with only one of the considered crack initiation points are visualized.

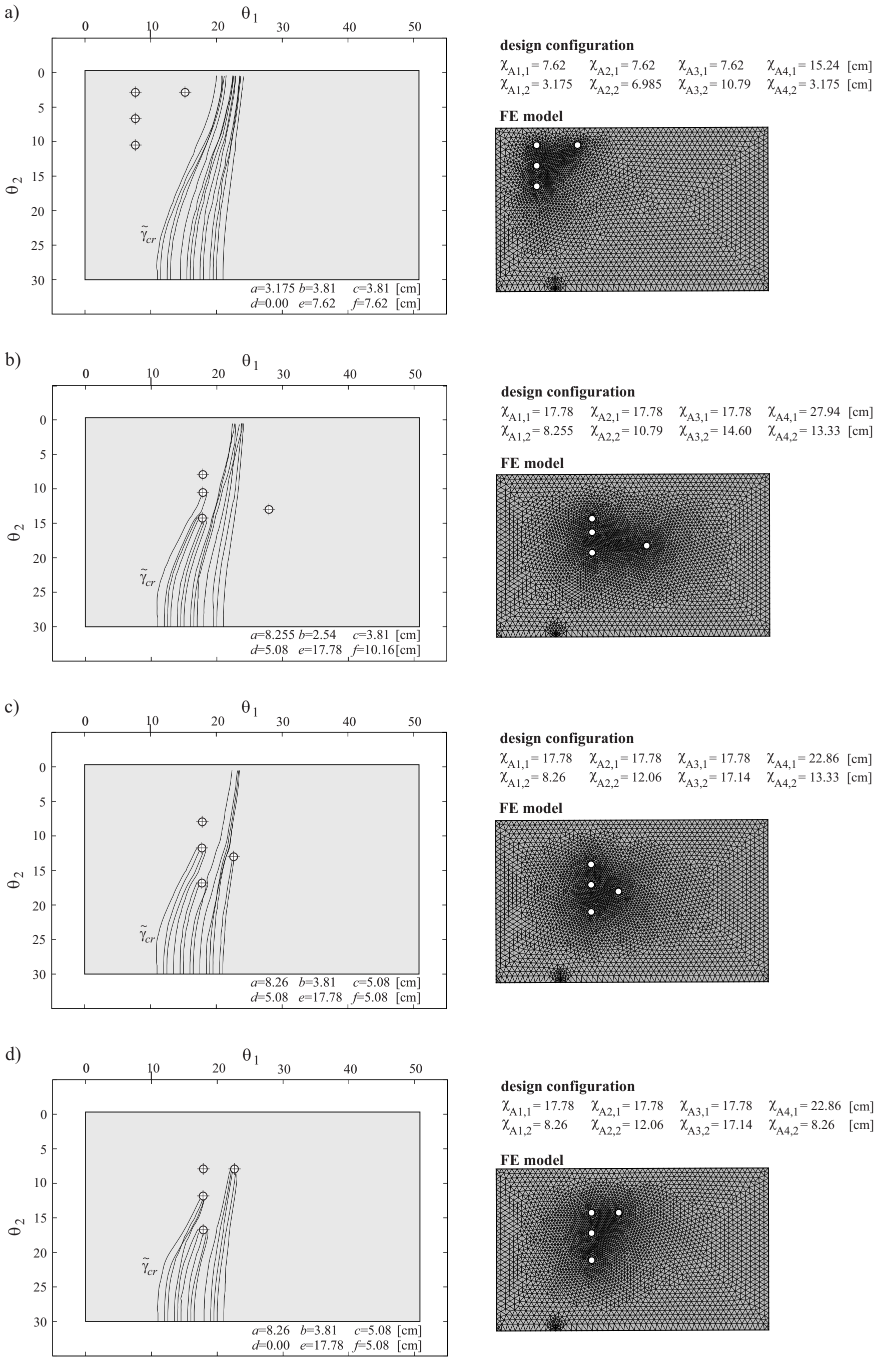

Figure 8. Fuzzy crack propagation curve for designs in subsequent generations of the optimization algorithm.

7. Conclusions

In the present contribution, a method for a fail-safe design optimization is presented, which is numerically realized as a coupled approach of optimization, fuzzy analysis and fracture mechanical analysis. For the investigations on safe failure, the uncertainties within the crack initiation and growth process are analyzed. The modelling of nonstochastic properties of the uncertain crack initiation position in a substructure belonging to the system of coupled substructures, succeeds by the application of the fuzzy set theory and the possibility theory framework. By taking into account the possibility of the occurrence of diverse crack propagation paths, the optimal configuration of the crack arresters can be identified in a systematic way by means of the genetic algorithm based design optimization procedure. The introduced approach enables the improvement of the prevention from undesired crack growth and damage escalation as well as contributes to the enhancement of structural durability and safety.

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